MULTIPLE CHOICE. Circle the correct answer from [a]–[h]. **SCORE: _____ / 3 PTS**

Which of the following expressions are defined ?

[i] $\vec{u} \cdot (\vec{v} \times \vec{w})$ YES [ii] $\vec{u} \times (\vec{v} \times \vec{w})$ YES [iii] $(\vec{u} \cdot \vec{v}) \times \vec{w}$ NO

[a] none are defined **[b]** only [i] is defined **[c]** only [ii] is defined **[d]** only [iii] is defined

[e] only [i] & [ii] are defined **[f]** only [i] & [iii] are defined **[g]** only [ii] & [iii] are defined **[h]** all are defined

Consider the sphere $x^2 + y^2 + z^2 - 4x + 6y + 10z + 29 = 0$. **SCORE: ____ / 5 PTS**

[a] Find the radius of the sphere.

$x^2-4x+4 + y^2+6y+9 + z^2+10z+25 = -29+4+9+25$ (1)

$(x-2)^2 + (y+3)^2 + (z+5)^2 = 9$ (1/2)

$r = \sqrt{9} = 3$ (1)

[b] Find the equation of the yz – trace of the sphere, and describe the trace in words.

$(0-2)^2 + (y+3)^2 + (z+5)^2 = 9$

$(y+3)^2 + (z+5)^2 = 5$ (1)

(1/2) CIRCLE, WITH CENTER $(0, -3, -5)$ (1/2)

RADIUS $\sqrt{5}$ (1/2)

A parallelepiped has adjacent edges $\vec{u}$, $\vec{v}$ and $\vec{w}$. **SCORE: ____ / 3 PTS**

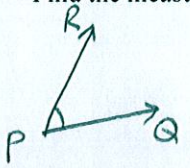
If $\vec{u} = 3\vec{i} + 2\vec{j} - 4\vec{k}$ and $\vec{v} \times \vec{w} = 5\vec{j} + 6\vec{k}$, find the volume of the parallelepiped.

$|\vec{u} \cdot (\vec{v} \times \vec{w})| = |3(0) + 2(5) - 4(6)| = |-14| = 14$

(2) (1)

Let P be the point $(-5,-2,3)$. Let Q be the point $(3,2,-1)$. Let R be the point $(-3,4,-2)$. **SCORE: ____ / 19 PTS**

[a] Find the measure of angle RPQ. **Give your final answer in degrees, rounded to 1 decimal place.**



$\vec{PR} = \langle 2,6,-5 \rangle$ (1)

$\vec{PQ} = \langle 8,4,-4 \rangle$ (1)

$\theta = \cos^{-1}\dfrac{\vec{PR}\cdot\vec{PQ}}{\|\vec{PR}\|\,\|\vec{PQ}\|} = \cos^{-1}\dfrac{60}{\sqrt{165}\sqrt{96}} \approx 40.6^\circ$ (1) (1) (1) (1)

[b] Find the general form ($Ax+By+Cz+D=0$) of the equation of the plane passing through P, Q and R.

$\vec{PR}\times\vec{PQ} = \langle -24--20, -(-8--40), 8-48 \rangle$

$= \langle -4,-32,-40 \rangle$ (1) $\rightarrow$ USE $\vec{n} = -\frac{1}{4}\langle -4,-32,-40 \rangle$

$= \langle 1,8,10 \rangle$

$1(x-3)+8(y-2)+10(z-(-1))=0$

$x+8y+10z-9=0$ (1)

[c] Find parametric equations of the line passing through R and parallel to the line $6-x=\dfrac{z+7}{3}$, $y=-5$.

$\vec{d} = \langle -1,0,3 \rangle$

$x = -3-t$
$y = 4$
$z = -2+3t$ (1) (1)

[d] Find a unit vector perpendicular to both $\vec{PQ}$ and $\vec{PR}$.

$\dfrac{1}{\|\vec{PR}\times\vec{PQ}\|}(\vec{PR}\times\vec{PQ}) = \dfrac{1}{\|-4\langle 1,8,10\rangle\|}(-4\langle 1,8,10\rangle)$ (1) FROM [b]

$= \dfrac{1}{4\,\|\langle 1,8,10\rangle\|}(-4\langle 1,8,10\rangle)$

$= \dfrac{1}{\sqrt{165}}\langle -1,-8,-10\rangle = \left\langle -\dfrac{\sqrt{165}}{165}, -\dfrac{8\sqrt{165}}{165}, -\dfrac{2\sqrt{165}}{33} \right\rangle$ (2)

[e] Find symmetric equations of the line passing through Q and perpendicular to the plane $9x-4y+8=0$.

$\vec{d} = \langle 9,-4,0 \rangle$

$\dfrac{x-3}{9} = \dfrac{y-2}{-4}$, $z=-1$

$\dfrac{x-3}{9} = \dfrac{2-y}{4}$ (1), $z=-1$ (1)

[f] If a force represented by the vector $\vec{PR}$ is applied to an object which moves from P to Q, find the work done.

$\vec{PR}\cdot\vec{PQ} = 60$ (1) FROM [a]

[g] Find the area of the triangle with vertices P, Q and R.

$\frac{1}{2}\|\vec{PR}\times\vec{PQ}\| = \frac{1}{2}\,4\sqrt{165} = 2\sqrt{165}$ (1) (1)

FROM [d]